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Math 118 - Fall 2025 - Common Final Exam

Print name:____

Section number:__________ **Instructor's name:**____________________________

Directions:

- **Do not open your exam until you are instructed to do so by your instructor.**
- Fill in your name, your instructor's name, and section number. If you don't know your section number, write the day and time that your class normally meets (example: MWF 8:15-9:05).
- This exam has 14 questions. Once the exam has begun, please check to make sure that your exam is complete.
- It will be graded out of 100 points.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- A formula sheet has been provided with this exam. You may not refer to any other notes during the exam.
- You may use a calculator which does not allow internet access. The use of any notes or electronic devices other than a calculator is prohibited.
- **Unless otherwise stated, round any constants to two decimal places if necessary.**

Good luck!

Question:	1	2	3	4	5	6	7	8
Points:	8	7	10	7	8	6	8	6
Score:								
Question:	9	10	11	12	13	14		Total
Points:	8	6	5	10	6	5		100
Score:								

1. (8 points) A social media influencer starts with 800 followers on her platform in January 2024. Recall that a linear function has a general form of $f(t) = a + bt$ and an exponential function has a general form of $f(t) = a \cdot b^t$.

   (a) If the influencer gains followers at a steady rate of 75 followers per month, find a formula for the function $f(t)$, the number of followers t months after January 2024.

   $\square\, f(t) = 800 + 75t$ ■ **(2 pts.)**

   (b) If her follower count is growing by 8% per month, find a formula for the function $f(t)$, the number of followers t months after January 2024.

   $\square\, f(t) = 800(1.08)^t$ ■ **(2 pts.)**

   (c) Under the assumptions stated in part (b), find how many months it will take for her follower count to reach 3000. Round to the nearest whole number.

$$\square\, 800(1.08)^t = 3000 \qquad \textbf{(1 pt.)}$$

$$(1.08)^t = \frac{15}{4}$$

$$t\ln(1.08) = \ln\frac{15}{4}$$

$$t = \frac{\ln(3.75)}{\ln(1.08)} \approx 17.17 \qquad \textbf{(3 pts.)}$$

   It takes about 17 months for the follower count to reach 3000. ■

2. (7 points) A startup's revenue grows by a factor of 1.15 each year. Find the doubling time for the revenue (in years). Give your answer to two decimal places.

   $\square$ With revenue being $R(t)$, we seek at what time we have $R(t) = 2R_0$:

$$R(t) = R_0(1.15)^t$$

$$2R_0 = R_0(1.15)^t \qquad \textbf{(3 pts.)}$$

$$\ln 2 = t\ln 1.15$$

$$t = \frac{\ln 2}{\ln 1.15} \approx 4.96 \qquad \textbf{(3 pts.)}$$

   The doubling time is about 4.96 years. ■ **(1 pt.)**

3. (10 points) Jordan deposits \$8,000 into a savings account that offers a nominal annual rate of 3.6%.

(a) Find the account balance after 4 years for the following compounding options. Round your answers to the nearest dollar.

i. Compounded annually.

$$\square\, B(4) = 8000\left(1 + \frac{0.036}{1}\right)^{(1)(4)} = \cdots = \$9215.71 = \$9216 \blacksquare \quad \textbf{(2 pts.)}$$

ii. Compounded monthly (12 times per year).

$$\square\, B(4) = 8000\left(1 + \frac{0.036}{12}\right)^{(12)(4)} = \cdots = \$9237.08 = \$9237 \blacksquare \quad \textbf{(3 pts.)}$$

iii. Compounded continuously.

$$\square\, B(4) = 8000e^{0.036(4)} = \cdots = \$9239.07 \doteq \$9239 \blacksquare \quad \textbf{(2 pts.)}$$

(b) If Jordan plans to keep the money in the account for 30 years, by what percent does the investment increase over the 30-year period if compounded annually? Round your answer to the nearest hundredth of a percent.

$$\square\, P(30) = P_0(1 + 0.036)^{30} = P_0(2.8893) = P_0 + 1.8893P_0$$

So the investment has increased by 188.93%. ■ **(3 pts.)**

4. (7 points) Consider the function $P(t) = 120e^{-0.09t}$, $t \geq 0$.

(a) Does $P(t)$ model exponential growth or decay? Explain briefly.

☐ Because $k < 0$ in the form e^{kt}, we have decay. ■ **(1 pt.)**

(b) What is the initial value $P(0)$?

☐ $P_0 = 120$ ■ **(1 pt.)**

(c) What is the continuous growth/decay rate as a percentage?

☐ The value $k = -0.09$ means we have a decay rate of 9%. ■ **(2 pts.)**

(d) Rewrite $P(t)$ in the form $P(t) = ab^t$ with $b > 0$.

$$\square\, P(t) = 120e^{-0.09t} = 120\left(e^{-0.09}\right)^t = 120(0.914)^t \blacksquare$$ **(3 pts.)**

5. (8 points) On March 1 at midnight low tide at a harbor is at 0.5 ft, and the first high tide that day occurs at 6 a.m. with height 8.5 ft.

(a) Find a sinusoidal model for the height at time t hours since midnight that fits the data, where t is hours after midnight. Show how you determine amplitude, period, and midline.

(1 pt.)
- The variation of height $h(t)$ (0.5 ft to 8.5 ft) covers 8 feet altogether, so the amplitude is $A = 4$.

(1 pt.)
- The midline is then at $h = 0.5 + 4$, or equivalently $8.5 - 4$; either way, the midline is at $h = 4.5$.

(1 pt.)
- A full tide cycle (low to high to low) will take 12 hours, so the period is 12 hours. With that $2\pi/B = 12$ gives $B = \pi/6$.

(2 pts.)
- Because the tide cycle starts low and then goes high, it's best to use an upside down cosine with all this data:

$$h(t) = -4\cos\left(\frac{\pi}{6}t\right) + 4.5 \blacksquare$$

(b) Write an equation to solve for the first time after midnight that the tide reaches 3 ft, and give the solution by using an inverse trig function. Then evaluate the time to the nearest hour.

☐ We have $h(t) = 3$ at:

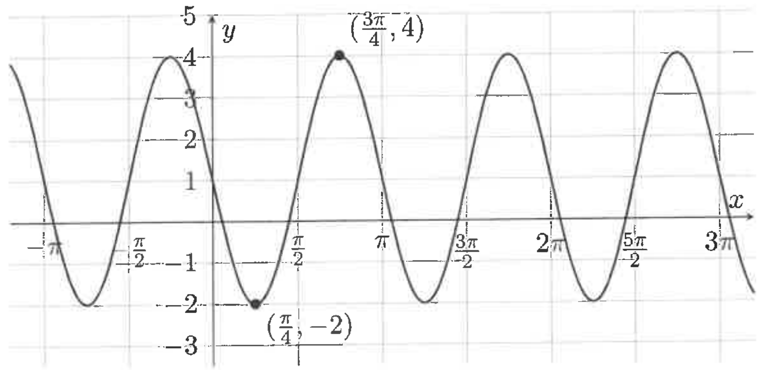
$$\begin{aligned} -4\cos\left(\frac{\pi}{6}t\right) + 4.5 &= 3 \\ \cos\left(\frac{\pi}{6}t\right) &= 0.375 \\ \frac{\pi}{6}t &= \cos^{-1}(0.375) \\ t &= \frac{6}{\pi}\cdot\cos^{-1}(0.375) \approx 2.266 \end{aligned}$$

So the tide reaches 3 feet at about 2AM. ■ **(3 pts.)**

6. (6 points) The graph of a trigonometric function is shown below. Write an equation of the form

$$y = A\sin(Bx + C) + D \quad \text{or} \quad y = A\cos(Bx + C) + D$$

that represents the graph. State the amplitude, period and midline. Show your work.



Amplitude = 3

Period = π

(3 pts.) Midline = 1

Let's set this as an inverted cosine, so that (so far) , $f(x) = -3\cos(2x + C) + 1$.

Starting with the inverted cosine, the horizontal shift is to the right by $\pi/4$, so to find C:

$$2t + C = 0 \rightarrow 2\left(\frac{\pi}{4}\right) + C = 0 \rightarrow C = -\frac{\pi}{2}$$

(3 pts.) Equation: $f(x) = -3\cos\left(2x - \frac{\pi}{2}\right) + 1$ OR $f(x) = -3\sin(2x) + 1$

7. (8 points) Let α be an angle in quadrant II with $\cos\alpha = -\frac{4}{5}$.

(a) Find $\sin\alpha$ and $\tan\alpha$. Provide an exact answer.

$$\square \ \sin\alpha = \frac{3}{5} \quad ; \quad \tan\alpha = -\frac{3}{4} \ \blacksquare$$ **(4 pts.)**

(b) Compute $\sin(\alpha - \frac{\pi}{6})$ using sum/difference identities. Provide an exact answer. **(2 pts.)**

$$\square \ \sin\left(\alpha - \frac{\pi}{6}\right) = \sin\alpha\cos\frac{\pi}{6} - \cos\alpha\sin\frac{\pi}{6}$$
$$= \frac{3}{5}\cdot\frac{\sqrt{3}}{2} - \left(-\frac{4}{5}\right)\cdot\frac{1}{2}$$
$$= \frac{3\sqrt{3}+4}{10} \ \blacksquare$$

(2 pts.)

8. (6 points) Evaluate each of the following exactly. Show all work and include a right triangle diagram when helpful.

(a) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$ or $45°$ **(2 pts.)**

(b) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$ or $150°$ **(2 pts.)**

(c) $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$ or $60°$ **(2 pts.)**

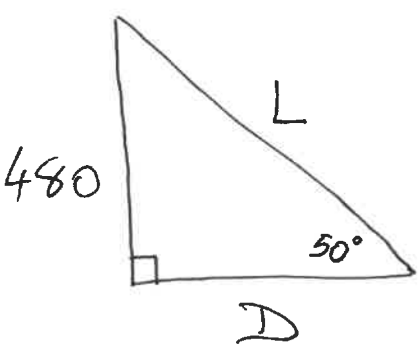
9. (8 points) A radio tower of height 480 ft is secured by a cable attached to the top and anchored to the ground. The cable makes an angle of 50° with the ground.

(a) How long must the cable be? Give your answer to two decimal places.

☐ $\sin(50°) = \frac{480}{L} \rightarrow L \approx 626.60$ ft $\blacksquare$ **(4 pts.)**

(b) How far from the base should the anchor be placed? (Distance from tower base to anchor point.)

☐ $\tan(50°) = \frac{480}{D}$ $D \approx 402.77$ ft $\blacksquare$ **(4 pts.)**



10. (6 points) Let $f(x) = 4x + 1$, $g(x) = 2^x$, and $h(x) = \ln(x)$.

(a) Compute $f(g(3))$.

$\square\, f(g(3)) = f(2^3) = f(8) = 4(8) + 1 = 33$ ■ **(2 pts.)**

(b) Write an expression for $h(g(f(x)))$. Do not simplify.

$\square\, g(f(x)) = 2^{4x+1}$ so $h(g(f(x))) = \ln 2^{4x+1} = (4x+1)\ln 2$ ■

(4 pts.) *(optional)*

11. (5 points) Let $G(x) = \dfrac{3}{\ln(2x+5)}$. Decompose G as $G(x) = f(u(x))$ with u the inside function.

$\square\, G(x) = f(u(x))$ with $f(x) = \dfrac{3}{\ln x}$ **(3 pts.)** and $u(x) = 2x + 5$ **(2 pts.)**

<u>OR</u>

$G(x) = f(u(x))$ with $f(x) = \dfrac{3}{x}$ **(2 pts.)** and $u(x) = \ln(2x+5)$ ■ **(3 pts.)**

12. (10 points) A town has population modeled by $P(t) = 900(1.04)^t$, where t is years since 2022.

(a) Evaluate $P(5)$ and round to the nearest whole number. Explain what this number represents in a sentence.

$\square\, P(5) = 900(1.04)^5 \approx 1095$ **(1 pt.)**

The population in 2027 is about 1,095. ■ **(2 pts.)**

(b) Find the inverse function $P^{-1}(y)$ in terms of y (exact form).

$\square$ With $P = 900(1.04)^t$,

$$\frac{P}{900} = 1.04^t$$

$$\ln\frac{P}{900} = t \cdot \ln 1.04$$

$$t = f^{-1}(P) = \frac{1}{\ln 1.04}\ln\frac{P}{900}$$

(4 pts.)

(c) Use part (b) to find how many years after 2022 the population will reach 1200. Round to the nearest whole year.

$$\square\, t = P^{-1}(1200) = \frac{1}{\ln 1.04}\ln\frac{1200}{900} = \frac{1}{\ln 1.04}\ln\frac{4}{3} \approx 7$$

The population reaches 1200 in about 7 years after 2022, i.e. about 2029. ■ **(3 pts.)**

13. (6 points) Perform the following coordinate conversions and give exact answers when possible.

(a) Convert Cartesian $(x, y) = (-\sqrt{3}, 1)$ to polar coordinates (r, θ) with $0 \le \theta < 2\pi$.

$$\square r^2 = x^2 + y^2 = 3 + 1 = 4 \quad \rightarrow \quad r = 2 \qquad \textbf{(1 pt.)}$$

$$\tan\theta = \frac{y}{x} = -\frac{1}{\sqrt{3}} \quad \rightarrow \quad \theta = \frac{5\pi}{6} \qquad \textbf{(2 pts.)}$$

$$(r, \theta) = \left(2, \frac{5\pi}{6}\right) \blacksquare$$

(b) Convert polar coordinates $(r, \theta) = (6, -\frac{\pi}{3})$ to Cartesian (x, y).

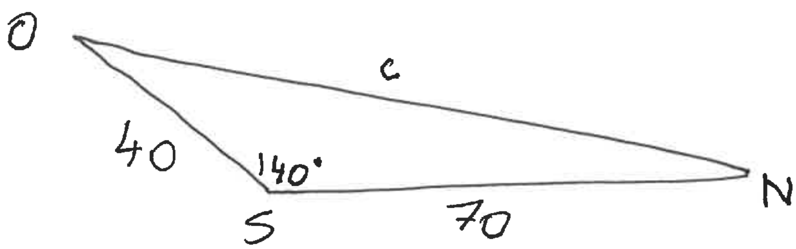
$$\square x = r\cos\theta = 6\cos\left(-\frac{\pi}{3}\right) = 6\left(\frac{1}{2}\right) = 3 \qquad \textbf{(1 pt.)}$$

$$y = r\sin\theta = 6\sin\left(-\frac{\pi}{3}\right) = 6\left(-\frac{\sqrt{3}}{2}\right) = -3\sqrt{3} \qquad \textbf{(2 pt.)}$$

$$(x, y) = (3, -3\sqrt{3}) \blacksquare$$

14. (5 points) Oli and Nora run from the same point with an angle of 140° between their paths. Oli runs 40 m, and Nora runs 70 m.

(a) Draw a clear diagram showing the situation.



(2 pts.)

(b) How far apart are Oli and Nora after running? Round your answer to the nearest meter.

$$\square c^2 = 70^2 + 40^2 - 2(70)(40)\cos(140) \qquad \textbf{(1 pt.)}$$
$$= 6500 - 5600\cos(140)$$
$$\approx 10789.85$$
$$c \approx 103.87$$

They are about 104 meters apart. ■ **(2 pts.)**

Rules of Exponents

$a^b a^c = a^{b+c}$

$(a^b)^c = a^{b\cdot c}$

$a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$

$a^{-b} = \dfrac{1}{a^b}$

$\dfrac{1}{a^{-b}} = a^b$

Exponential and Logarithm Formulas

Linear Function: $Q(t) = mt + b$

Exponential Function: $Q(t) = a \cdot b^t$

Continuous Exponential Function: $Q(t) = a \cdot e^{kt}$

Simple Interest: $B = P(1 + r)^t$

Compound Interest: $B = P\left(1 + \dfrac{r}{n}\right)^{nt}$

Logarithms: $b^y = x \leftrightarrow log_b(x) = y$

Trigonometry Formulas

1 radian $= \dfrac{180}{\pi}$ degrees and 1 degree $= \dfrac{\pi}{180}$ radians

$\sin(\theta) = \dfrac{opp}{hyp} = \dfrac{y}{r}$ $\quad\cos(\theta) = \dfrac{adj}{hyp} = \dfrac{x}{r}$ $\quad\tan(\theta) = \dfrac{opp}{adj} = \dfrac{y}{x} = \dfrac{\sin(\theta)}{\cos(\theta)}$

$\csc(\theta) = \dfrac{1}{\sin(\theta)} = \dfrac{r}{y}$ $\quad\sec(\theta) = \dfrac{1}{\cos(\theta)} = \dfrac{r}{x}$ $\quad\cot(\theta) = \dfrac{1}{\tan(\theta)} = \dfrac{x}{y} = \dfrac{\cos(\theta)}{\sin(\theta)}$

Sum and Difference Formulas:

$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$

$\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$

$\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

$\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$

Pythagorean Identities: $\sin^2(\theta) + \cos^2(\theta) = 1$ $\quad\tan^2(\theta) + 1 = \sec^2(\theta)$ $\quad 1 + \cot^2(\theta) = \csc^2(\theta)$

Even-Odd Identities: $\sin(-x) = -\sin(x)$ and $\cos(-x) = \cos(x)$ and $\tan(-x) = -\tan(x)$

Other identities: $\sin(\theta) = \sin(\pi - \theta)$, $\cos(\theta) = \cos(2\pi - \theta)$

General form for sine and cosine: $f(t) = A\sin(Bt) + k$ and $f(t) = A\cos(Bt) + k$

General form with horizontal shift: $f(t) = A\sin(B(t - h)) + k$ and $f(t) = A\cos(B(t - h)) + k)$

Period for sine and cosine: $P = \dfrac{2\pi}{|B|}$ or $PB = 2\pi$. Amplitude $= |A| = \dfrac{max - min}{2}$. Midline: $y = k$, where $k = \dfrac{max + min}{2}$

Law of Sines: $\dfrac{\sin(A)}{a} = \dfrac{\sin(B)}{b} = \dfrac{\sin(C)}{c}$

Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos(C)$

Arc Length: $s = r\theta$

Inverse Trig Functions
$\theta = \cos^{-1}(y)$ provided that $y = \cos(\theta)$ and $0 \leq \theta \leq \pi$
$\theta = \sin^{-1}(y)$ provided that $y = \sin(\theta)$ and $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$
$\theta = \tan^{-1}(y)$ provided that $y = \tan(\theta)$ and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

Polar coordinates conversions
$r^2 = x^2 + y^2$, $\tan(\theta) = \frac{y}{x}$, $x = r\cos(\theta)$, $y = r\sin(\theta)$

The Unit Circle